**Quick Introduction to Complex Numbers**

In order to describe how the Mandelbrot Set is generated, we have to begin with a quick summary/review of "complex numbers".  (Sometimes we call these numbers "imaginary numbers", but that is silly since *all* numbers are imaginary.)

Start with the numbers that we are used to working with:  the Real numbers.  Then add to this set a brand new number, *i*, which has the property that: ***i*2*=* -1**

If you start combining this new number with all of the Real ones by using addition, subtraction, multiplication and division, you'll be able to generate lots and lots of new numbers that look like polynomials in terms of *i*.  For example:

155 - 13 *i*10 + 72 *i*17

Taking into account the fact that ***i*2*=* -1**, you'll see that you can always *simplify* a crazy-looking polynomial like the one above.  For example, the polynomial above is the same thing as:

155 - 13 *(i*2)5 + 72 (*i*2)8(*i*)  =  155 - 13 *(*-1)5 + 72 (-1)8(*i*)  =   155 + 13 + 72*i* =   168 + 72*i*

In fact:  All polynomials that are built by combining the real numbers with this new number, *i*, can be simplified to the form:  *a* + *bi* (where *a* and *b* are real numbers).  We call the collection of such polynomials the "Complex Numbers".

**Real and Imaginary Components**

Consider a typical complex number:  47.9 - 62.55*i*

The first term, 47.9, is called the "*real component*" of the number; the second term, -62.55*i*,  is called the "*imaginary component*".

**Arithmetic with Complex Numbers**

*Adding* complex numbers together is easy.  We just have to add the like terms.

For example, let's add the number -2 + 7*i* to the number 9 - 4*i:* (-2 + 7*i*)  +  (9 - 4*i*)*=  (-2+9) + (7i - 4i) = (-2+9)+(7-4)i =* 7 + 3*i*

Another example, let's add the number 2 - 5*i* to the number -3 - 4*i:* (2 - 5*i*)  +  (-3 - 4*i*)*=  (2-3) + (-5i - 4i) = (2-3)+(-5-4)i =* -1 -9*i*

*Subtracting* complex numbers is just as easy as adding.  For example:

For example, lets subtract the number 9-4i from the  number -2+7i: (-2 + 7*i*)  -  (9 - 4*i*)*=(-2-9) + (7i--4i) = (-2-9)+(7+4)i =* -11 + 11*i*

Another example, lets subtract the number 2+4i from the  number -2-4i: (-2 - 4*i*)  -  (2 + 4*i*)*=(-2-2) + (-4i-4i) = (-2-2)+(-4-4)i =* -4 - 8*i*

*Multiplying* complex numbers is a little harder, but not too bad.  We have to multiply two binomials together, and then combine like terms.  **Remember that any time you see i2 you can replace it with -1**.

As an example, let's multiply the number -2 + 5*i* by the number 9 - 4*i*

(-2 + 5*i*) (9 - 4*i*) *=* (-2)(9) + (-2)(-4*i*) + (5*i*)(9) + (5*i*)(-4*i*)  =  -18  +  8*i*  + 45*i*  - 20*i*2  =  -18  + 53*i*  + 20  =  2 + 53*i*

As another example, let's multiply the number 4 + 2*i* by the number 3 + 2*i*

(4 + 2*i*) (3 + 2*i*) *=* (4)(3) + (4)(*2i*) + (2*i*)(3) + (2*i*)(2*i*)  =  12  +  8*i*  + 6*i*  + 4*i*2  =  12  + 14*i*  - 4  =  8 + 14*i*

*Dividing* complex numbers is a huge pain!  before giving a concrete example, we'll just give the formula for dividing the number *a + bi* by the number *c + di.*

  (a + bi)        (ac + bd)        (bc - ad)

  --------  =     ----------   +   --------- *i*

  (c + di)        (c2 + d2)        (c2 + d2)

   So for example, if we want to divide 16-2i by 3-2i we would have:

   (16 - 2i)          (16\*3 + -2\*-2)         (-2\*3-(16\*-2))            (48 +4)        (-6 + 32)     52          26

    --------  =     ----------------   +   ----------------- *i   =*  ----------   +   ---------- *i =*  -----   +   ----- *i = 4+2i*

    (3 - 2i)           (32 + (-2)2)             (32 + (-2)2)         (9 + 4)          (9+ 4)         13         13

 Or for another example, if we want to divide -8+27i by 2+3i we would have:

 (-8 + 27i)    (-8\*2) + (27\*3)        (27\*2) -(-8\*3)           -16 + 81      54 + 24        65      78

  --------  =   ------------------- + ------------------- i = ------------+ ---------- i = ---- + ----- i = 5+6i

  (2 + 3i)          (22 + 32)                (22 + 32)             4+9           4+9         13      13

**The *"Norm"* of a Complex Number**

The *norm* of a complex number is a Real number that measures how large the complex number is.  We define the "norm" of a complex number *a + bi* to be equal to *sqrt(a2 + b2).*  [The square root of *a* squared plus *b* squared.]

For example, norm (3 + 4*i)* = sqrt(32 +42) = sqrt(9+16) = sqrt(25) = 5.

And another example, norm ( 5-12i) = sqrt(52 +(-12)2) = sqrt(25+144) = sqrt(169) = 13.

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